Acoustic-optical Image Denoising Using Alpha-Stable Multivariate Shrinkage Function in the Contourlet Domain

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Abstract

We describe a method for removing acoustic-optical image noise, based on a statistical model of the decomposed contourlet coefficients. This method proposes an alpha-stable multivariate shrinkage (MS) probability density function to model neighborhood coefficients. Then, contourlet according to the proposed PDF model, we design a maximum a posteriori (MAP) estimator, which relies on a Bayesian statistics representation for the contourlet coefficients of acoustic-optical images. There are two obvious virtues of this method. Firstly, contourlet transform decomposition is prior to curvelettransform by using ellipse sampling grid. Secondly, non-Gaussian multivariate shrinkage model is more effective in presentation of the acoustic-optical image contourlet coefficients. Some comparisons with the best available results will be presented in order to illustrate the effectiveness of the proposed method.

Keywords: α -stable Distribution, contourlet transform, image denoising, statistical modeling, deep-sea signal processing

1. Introduction

Typically, acoustics and videos are the most suitable ways to create an understanding of the geometry and appearance of the underwater environment [15]. However, in the deep sea, the nature light is not available (the nature light approximately 200 meters below the surface of the ocean). Even if the artificial light is applied on Autonomous Underwater Vehicles (AUV) or Remotely Operated Vehicles (ROV), the visible range is very limited, such as the wavelength of the light is short and the turbid water affection is highly. For these reasons, sonar systems are widely used to obtain acoustic images of the seabed.

Recently, an acoustic-optical camera DIDSON [1] can acquire high-resolution ultrasonic images with imaging device is proposed. Which is useful for offshore guidance, fish monitor, hydrothermal vents and methane hydrate survey. Despite the quality of the video sequences is image likely, it still has shortcomings compared with normal optical cameras. With limitation of sight range, there are low signal-to-noise ratio, and low resolution et. al [16], [17].

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To some extent, the above drawbacks can be solved by image denoising. Some research has addressed the development of statistical models of wavelet coefficients of images [2], [3], [4]. However, the major drawback for wavelets in two-dimensions or higher is their limited ability in capturing directional information. To overcome this deficiency, some researchers have recently considered multiscale and directional representations that can well capture the geometrical structures such as wedgelets [5], bandelets [6], curvelets [7], [8] and contourlets [9], [12]. Using curvelet transform for decomposition, blocks must be overlapped together to avoid the boundary effect. Therefore, redundancy is higher in this implementation algorithm. Additionally, the curvelet transform is based on ridgelet transform, which the key step is the Cartesian to polar conversion. It is hard to Cartesian-Polar conversion. Luckily, the contourlet transform is a "true" two-dimensional transform that can capture the intrinsic geometrical structure. Contourlet transform represents better the salient features of the image such as edges, lines, curves, and contours, than wavelet transform because of its anisotropy and directionality.

The organization of this paper is as follows: In Section 2, the contourlet-based alpha-stable multivariate shrinkage (\square MS) method will be discussed. We will demonstrate an image denoising algorithm to exploit interscale dependencies, and also apply the contourlet \square MS model in acoustic-optical image and natural image denoising in Section 3. Finally, a conclusion is presented in Section 4.

2. Proposed Moedl

In this section, we propose a new model for non-Gaussian image denoising, which based on Bayesian MAP estimator rule. "g" is equally spaced in the samples of a real-valued image. n is i.i.d. normal random variables. The image with noise x can be expressed as

$$x = g + n$$
, (1)

In the contourlet domain, the problem can be formulated as

$$y = s + n, \qquad (2)$$

where y = (y1, y2, ..., yM) is the noise contourlet coefficient, s = (s1, s2, ..., sM) is the true coefficient, and n = (n1, n2, ..., nM) is the independent noise. The standard MAP estimator for Eq.(2) is

$$\hat{s}(y) = \arg\max p_{s|y}(s \mid y) \tag{3}$$

Using the Bayes rule, the Eq. (3) is equivalent to

$$\hat{s}(y) = \arg\max_{s} [p_{y|s}(y \mid s)p_{s}(s)]$$

=
$$\arg\max_{s} p_{n}(y-s)p_{s}(s)$$
(4)

The Eq.(4) is equivalent to

$$\hat{s}(y) = \arg\max[\log(p_n(y-s)) + \log(p_s(s))]$$
(5)

Thespherically-contouredzero-meand-dimensional BKF density is

$$p_{s}(s) = \frac{2}{(2\pi c)^{d/2} \Gamma(p)} \left(\frac{\sqrt{2c}}{\|s\|}\right)^{d/2-p} K_{1-p}\left(\sqrt{\frac{2}{c}} \|s\|\right)^{(6)}$$

where, $K_{\lambda}(u)$ is the modified Bessel function. *c* and *p* are the scale parameter and shape parameter. In this paper, we propose a simple non-Gaussian multivariate pdf to model the noise-free coefficients, considering the relationship among a coefficient, neighbors, cousins and parent.

$$p_{Y}(y) = \frac{\exp\left(\frac{\sigma_{n}^{2}}{c}\right)}{(2\pi c)\Gamma(p)} \times$$

$$\sum_{j=0}^{\infty} \left(-\frac{\sigma_{n}^{2}}{c}\right) \frac{(p-1)_{j}}{j!} \Gamma\left(p - \frac{d}{2} - j, \frac{\sigma_{n}^{2}}{c}; \frac{\|c\|^{2}}{2c}\right)$$
(7)

with*j* factor in both the numerator and denominator of the fraction. " σ n" is the standard deviation of the noise coefficients. In Ref. (9), the variables $y=sqrt(||y_i||^2+||y_i^{[p]}||^2+||y_i^{[c]}||^2)$, y_i and $y_i^{[p]}$ are dependent to each other, but the neighbors and cousins are independent to parent. The MAP estimator is used in this model. Maximizing the (7) for each component, we can get

$$y_i = \hat{s}_i - \sigma_n^2 \frac{d\log p_s(s)}{d\hat{s}_i} \tag{8}$$

where, *i*[I,d]. By the way, the property of the modified Bessel function of the second kind $K_{\lambda}(u)$ is

$$\frac{d}{du}\log K_{\lambda}(u) = \frac{\lambda}{u} - \frac{K_{\lambda+1}(u)}{K_{\lambda}(u)}$$
(9)

Then, the second term of Eq.(8) can be computed as

$$\frac{d}{d\hat{s}_{i}}\log p_{s}(s) = -\frac{\hat{s}_{i}}{\|\hat{s}\|} \sqrt{\frac{2}{c}} \frac{K_{d/2-p+1}(\sqrt{2/c} \|\hat{s}\|)}{K_{d/2-p}(\sqrt{2/c} \|\hat{s}\|)} (10)$$

Though the above eq., the MAP estimator can be formulated using Eq.(7) and Eq.(9), it gives

$$y_{i} = \hat{s}_{i} \left[1 + \frac{\sigma_{n}^{2}}{\|\hat{s}\|} \sqrt{\frac{2}{c}} \frac{K_{d/2-p+1} \left(\sqrt{2/c} \| \hat{s} \| \right)}{K_{d/2-p} \left(\sqrt{2/c} \| \hat{s} \| \right)} \right]$$
(11)

Then, we approximate the ||s|| as ||y||. The multivariate shrinkage function can be written as

$$\hat{s}_{i} \approx \frac{y_{i}}{\left[1 + \frac{\sigma_{n}^{2}}{\|y\|} \sqrt{\frac{2}{c}} \frac{K_{d/2-p+1}(\sqrt{2/c} \|y\|)}{K_{d/2-p}(\sqrt{2/c} \|y\|)}\right]}$$
(12)

where $\sigma_n = Median(y_i)/0.6748$, p=3/(Kurt(X)-3), c=Var(X)/p. *X* is the HH subband. Var(*X*) and Kurt(*X*) are the variance and kurtosis.

3. Experimental Results and Discussions

In this section, the examples of image denoising present the proposed models that wasdescribed in the previous section. There are the Windows XP, Core Duo2, 2.0GHz computer for simulations. In the first experiment, this study compares the Barbara image denoising, which results with Ref. [3], [7], [8], [9] and [12]. Through Human Visual System (HSV), the study found that the proposed method denoising ability obviously better than the other methods (see Figure 1). The sharpness of the denoised image is close to the original Barbara image. In Figure 2(a) plots the histogram of coefficients of the image Barbara, 512×512 pixels. The kurtosis of the distribution is 20. The Figure 2(b) shows that the coefficients have significantly non-Gaussian statistics, which that are best described by families of heavy-tailed distributions, $\alpha = 1.1311$.





Figure 1:Experimental results. (a)Barbara; (b) noisy (σn=20); (c) denoised with K-Sigma1; (d) denoised with K-Sigma2; (e) denoised with BLS-GSM; (f) denoised with GSCE; (g)denoised with Contourlet-HMT; (h) denoised with the proposed NGMS.



Figure 2:Marginal statistics of contourlet coefficients of the image "Barbara".
 (a)Histogram; (b) Statistics distributions of coefficients (α= 1.1311).

 Table 1. PSNR values of denoised images for Barbara with different noise levels, K-Sigma1[7], K-Sigma2 [12], BLS-GSM [3], GSCE [8], ContourletHMT [9], and proposed method.

Noise Level	Noisy Image	K-Sigma ₁	K-Sigma ₂	BLS-GSM	GSCE	Contourlet-HMT	Proposed
$\sigma_n=10$	28.19	28.47	29.28	33.08	31.71	31.40	34.63
$\sigma_n=20$	22.13	25.49	26.09	28.93	28.09	26.25	31.40
$\sigma_n=30$	18.64	24.56	24.33	26.78	26.07	23.01	28.31



Figure 3: Comparison of denoised results on Barbara in term of SSIM.

From the above table, the study finds that the proposed method is outperformed. The value of SSIM is highest than the others, being the higher is the better. The average improvement in term of PSNR values for test images is approximately 2.5dB. The average CPU processing time of the methods is $CPU_{K-Sigma1}=1.94s$, $CPU_{BLS-GSM}=31.33s$, $CPU_{GSCE}=16.11s$, $CPU_{K-Sigma2}=13.95s$, $CPU_{ContourletHMT}=18.93 s$, and $CPU_{Proposed}=15.41 s$. The processing time of the

proposed method is faster than [3], [9], and [8], for example, the Barbara image with abundant texture features. Besides, the study compares Mandrill (512X512 pixels) image with many detail characters. The denoised results are presented on Figure 4, Table 2, 3. The experiment also proved the effectiveness of the proposed algorithm.

Let x_i and y_i be the *i*-th pixel in the original image **x** and the distorted image **y** respectively. The *MSE* and *PSNR* between the two images are given by

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (x_i - y_i)^2, \qquad (13)$$

$$PSNR = 10\log_{10}\left(\frac{L^2}{MSE}\right)$$
(14)

In Ref. [13,14], a multi-scale SSIM method for image quality assessment is proposed. Input to signal A and B, let μ A, σ A and σ AB respectively as the mean of A, the variance of A, the covariance of A and B. The parameters of relative importance α , β , γ are equal to 1. The SSIM is given as follow:

$$SSIM(\mathbf{x}, \mathbf{y}) = \frac{(2\mu_A\mu_B + C_1)(2\sigma_{AB} + C_2)}{(\mu_A^2 + \mu_B^2 + C_1)(\mu_A^2 + \mu_B^2 + C_2)} (15)$$

where C_1 , C_2 are small constants.



Figure 4:Experimental results. (a)Mandrill; (b) noisy (σn=20); (c) denoised with K-Sigma1; (d) denoised with K-Sigma2; (e) denoised with BLS-GSM; (f) denoised with GSCE; (g)denoised with Contourlet-HMT; (h) denoised with the proposed NGMS.

 Table 2.PSNR values of denoised images for Mandrill with different noise levels, K-Sigma1[7], K-Sigma2 [12], BLS-GSM [3], GSCE [8], ContourletHMT [9], and proposed method.

Noise Level	Noisy Image	K-Sigma ₁	K-Sigma ₂	BLS-GSM	GSCE	Contourlet-HMT	Proposed
$\sigma_n=10$	28.16	24.40	25.72	30.21	26.89	23.97	33.45
$\sigma_n=20$	22.14	23.08	22.61	26.11	24.46	23.52	28.21
$\sigma_n=30$	18.62	22.22	21.20	24.02	23.05	22.54	25.44

 Table 3.SSIM values of denoised images for Mandrill with different noise levels, K-Sigma1[7], K-Sigma2 [12], BLS-GSM [3], GSCE [8], ContourletHMT [9], and proposed method.

Noise	Noisy	K-Sigma ₁	K-Sigma ₂	BLS-GSM	GSCE	Contourlet-HMT	Proposed
Level	Image						
$\sigma_n=10$	0.8407	0.7125	0.9015	0.9606	0.7879	0.8547	0.9837
$\sigma_n=20$	0.6430	0.6267	0.7742	0.8948	0.6943	0.8198	0.8268
$\sigma_n=30$	0.4932	0.5548	0.6638	0.8213	0.6103	0.7641	0.7467

As above, table 3 the K-Sigma methods are used the hard-thresholding rule to estimate the unknown wavelet, curvelet or contourlet coefficients, which tends to be unstable. The soft-threshold functions are proposed to improve it, but due to the shrinkage of large coefficients, it tends to have larger bias and decrease the precision of reconstructed signal. BLS-GSM performs a litter bad than the proposed method. The reason is the contourlet transform not only with nearly "completely" decomposition, that is the contourlet coefficients highly reflect the characteristics of the image, but also the coefficient relationships are well considered in the proposed NGMS model. In Ref. [9], the authors considered the coefficient relationships, due to the accuracy of the training HMT model. However, there is also exist some drawbacks. The drawbacks of GSCE are presented in the section 2.

In the third experiment, the study applies the proposed method in processing deep sea acoustic-optical 6 feet tire image in 100 meters depth Ocean. Through the experiment, the distribution of coefficients is also obeying the non-Gaussian distribution. Taking the proposed method for denoising, the value of PSNR is improved nearly 10dB. The Figure 5 displays that after denoising, the curves of the tire are more smoothly than the original one.



Figure 5:Deep sea acoustic-optical image denoising results. (a) original image, PSNR=23.87dB; (b) denoised by NGMS, PSNR=33.01dB.

In order to verify the ability and robustness of the proposed method in processing the acoustic-optical underwater images, the study tests 100 groups of images. The average PSNR are shown in Figure 6. From this Figure, it is found that the proposed method is well in donoising acoustic-optical images.



Figure	6:Afterdenoising Average PSNR	values of
	different methods in 100	samples.
	PSNR(K-Sigma1)=25.28	dB,
	PSNR(K-Sigma2)=26.16	dB,
	PSNR(BLS-GSM)=30.01	dB,
	PSNR(GSCE)=28.89	dB,
	PSNR(ContourletHMT)=23.28	dB,
	PSNR(Proposed)=31.36 dB	

4. Conclusions

This paper presents an effective and useful alpha-stable multivariate shrinkage (α MS) model for image denoising, which is based on the MAP estimation and the contourlet transform. We consider the dependencies among a coefficient and parents, and cousins in the α MS model. In order to show the effectiveness of the α MS estimator, three sets of examples are presented and compared with human visual system (HVS) and some well-defined mathematical frameworks. Experimental results demonstrate that the proposed α MS method outperforms the start-of-art methods.

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